HyPoRes: An Hybrid Representation System for ECC

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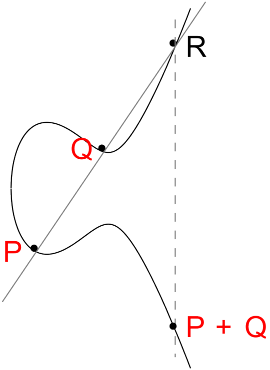
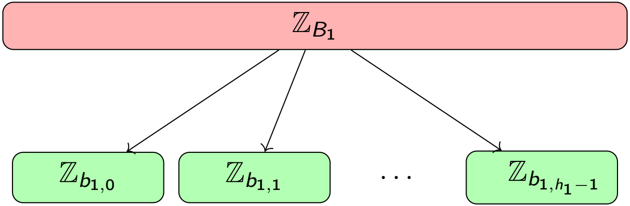
Elliptic Curve Cryptography



Point addition of two points over an EC defined in $\mathbb{R}$

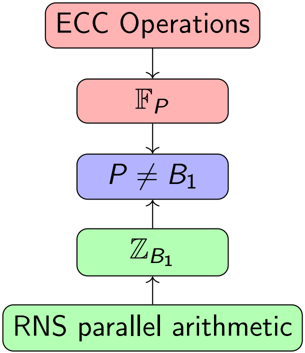
- Security based on the difficulty of computing n from $[n]P$ and P for curves defined over a finite field $\mathbb{F}_P$

Residue Number System



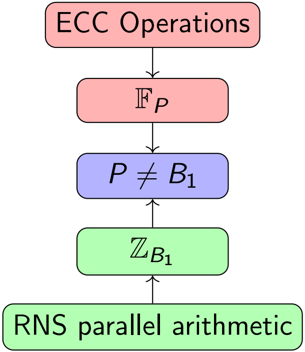
RNS breaks arithmetic modulo $B_1 = b_{1,0} \times \ldots \times b_{1,h_1-1}$ down to arithmetic modulo $b_{1,0}, \ldots, b_{1,h_1-1}$

Bridging the Gap



- **Montgomery Reduction**
  Maps operations in $\mathbb{F}_P$ to $\mathbb{Z}_{B_1}$ for any P with complexity of $\mathcal{O}(\log_2^2 P)$;
- **Hybrid-Positional Residue Number System (HPR)**
  Uses $P = B_1^n - \beta$ to reduce complexity to $\mathcal{O}(\log_2^{3/2} P)$.

Bridging the Gap



- **Montgomery Reduction**
  Maps operations in $\mathbb{F}_P$ to $\mathbb{Z}_{B_1}$ for any P with complexity of $\mathcal{O}(\log_2^2 P)$;
- **Hybrid-Positional Residue Number System (HPR)**
  Uses $P = B_1^n - \beta$ to reduce complexity to $\mathcal{O}(\log_2^{3/2} P)$.
  - Does not work for standardised primes

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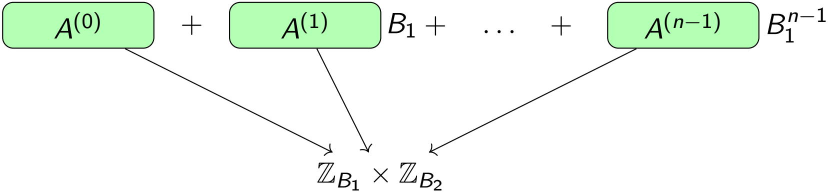
Montgomery Reduction

$$\begin{tikzcd}
A \sim P^2 \arrow[d] & \\
\begin{array}{c} Q_1 \text{ s.t. } B_1 | A + Q_1 P \\ Q_1 = -AP^{-1} \bmod B_1 \end{array} \arrow[r, "\mathcal{O}(h_1 h_2)"] & Q_2 = Q_1 \bmod B_2 \arrow[d] \\
Z_1 = Z_2 \bmod B_1 & Z_2 = \frac{A + Q_1 P}{B_1} \text{ small} \arrow[l, "\mathcal{O}(h_1 h_2)"']
\end{tikzcd}$$

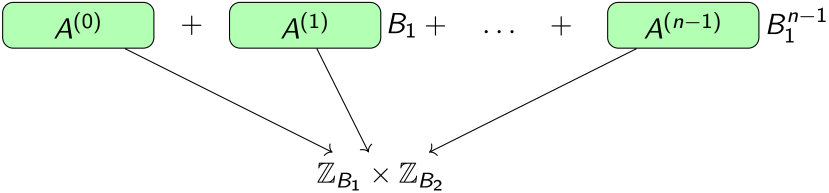
Complexity dominated by $\mathcal{O}(h_1 h_2)$ with $h_1 \sim h_2 \sim \log_2 P$

Hybrid-Positional Residue Number System



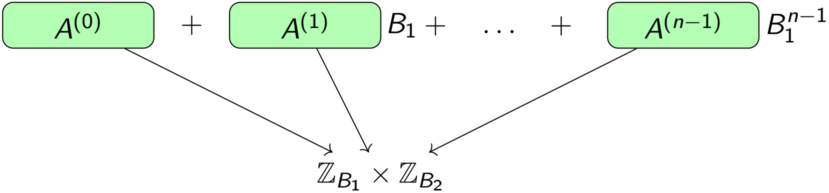
- $D = A \times C =$
  $D^{(0)} + D^{(1)} B_1 + \ldots + D^{(n-1)} B_1^{n-1} + D^{(n)} B_1^{n} + \ldots + D^{(2n-2)} B_1^{2n-2}$

Hybrid-Positional Residue Number System



- $D = A \times C =$
  $D^{(0)} + D^{(1)} B_1 + \ldots + D^{(n-1)} B_1^{n-1} + D^{(n)} B_1^{n} + \ldots + D^{(2n-2)} B_1^{2n-2}$
- For $P = B_1^n - \beta$:
  $D \equiv \left(D^{(0)} + \beta D^{(n)}\right) + \left(D^{(1)} + \beta D^{(n+1)}\right) B_1 + \ldots + D^{(n-1)} B_1^{n-1}$

Hybrid-Positional Residue Number System



- $D = A \times C = D^{(0)} + D^{(1)} B_1 + \dots + D^{(n-1)} B_1^{n-1} + D^{(n)} B_1^{n} + \dots + D^{(2n-2)} B_1^{2n-2}$
- For $P = B_1^n - \beta$: $D \equiv \left(D^{(0)} + \beta D^{(n)}\right) + \left(D^{(1)} + \beta D^{(n+1)}\right) B_1 + \dots + D^{(n-1)} B_1^{n-1}$
- Perform carry propagation to reduce the digits magnitude

Carry Propagation

$$
\begin{tikzcd}
D_1^{(i)} \bmod B_1 \arrow[r, "\mathcal{O}(h_1 h_2)"] & D_1^{(i)} \bmod B_2 \arrow[d] \arrow[dd, bend left] \\
C_1^{(i)} = C_2^{(i)} \bmod B_1 \arrow[dd] & C_2^{(i)} = \frac{D_2^{(i)} - D_1^{(i)}}{B_1} \arrow[l, "\mathcal{O}(h_1 h_2)"'] \arrow[dd, bend left] \\
 & D_2^{(i)} = D_1^{(i)} \\
D_1^{(i+1)} = D_1^{(i+1)} + C_1^{(i)} & D_2^{(i+1)} = D_2^{(i+1)} + C_2^{(i)}
\end{tikzcd}
$$

Complexity dominated by $\mathcal{O}(n^2(h_1 + h_2) + nh_1h_2)$ with $nh_1 \sim nh_2 \sim \log_2 P$

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Hybrid Polynomial-Residue Number System

$$
\begin{tikzcd}
 & & \mathbb{Z}_{B_1} \times \mathbb{Z}_{B_2} & & \\
\boxed{A^{(0)}} \arrow[urr] \arrow[drr] & + & \boxed{A^{(1)}}\, X \arrow[u] \arrow[d] & + \quad \ldots \quad + & \boxed{A^{(n-1)}}\, X^{n-1} \arrow[ull] \arrow[dll] \\
 & & a = \sum_{i=0} A^{(i)} \gamma^i \bmod P & &
\end{tikzcd}
$$

Hybrid Polynomial-Residue Number System

$$\mathbb{Z}_{B_1} \times \mathbb{Z}_{B_2}$$

$$\boxed{A^{(0)}} + \boxed{A^{(1)}} X + \ \dots \ + \boxed{A^{(n-1)}} X^{n-1}$$

$$a = \sum_{i=0} A^{(i)} \gamma^i \bmod P$$

$\boxed{\gamma \text{ is the n-}th\text{ root of a small value } \beta \text{ over } \mathbb{F}_P}$ $\Rightarrow X^n - \beta \cong 0$

Hybrid Polynomial-Residue Number System

$\boxed{\gamma \text{ is the n-}\textit{th}\text{ root of a small value } \beta \text{ over } \mathbb{F}_P} \Rightarrow X^n - \beta \cong 0$

- $D = A \times C = D^{(0)} + D^{(1)}X + \ldots + D^{(n-1)}X^{n-1} + D^{(n)}X^n + \ldots + D^{(2n-2)}X^{2n-2}$

Hybrid Polynomial-Residue Number System

$\boxed{\gamma \text{ is the n-}th\text{ root of a small value } \beta \text{ over } \mathbb{F}_P}$ $\Rightarrow X^n - \beta \cong 0$

- $D = A \times C = D^{(0)} + D^{(1)}X + \ldots + D^{(n-1)}X^{n-1} + D^{(n)}X^n + \ldots + D^{(2n-2)}X^{2n-2}$
- $D \equiv D - (D^{(n)} + \ldots + D^{(2n-2)}X^{n-2}) \times (X^n - \beta) \equiv \left(D^{(0)} + \beta D^{(n)}\right) + \left(D^{(1)} + \beta D^{(n+1)}\right) B_1 + \ldots + D^{(n-1)}B_1^{n-1}$

Hybrid Polynomial-Residue Number System

γ is the n-*th* root of a small value β over $\mathbb{F}_P$ $\Rightarrow X^n - \beta \cong 0$

- $D = A \times C = D^{(0)} + D^{(1)}X + \ldots + D^{(n-1)}X^{n-1} + D^{(n)}X^n + \ldots + D^{(2n-2)}X^{2n-2}$
- $D \equiv D - (D^{(n)} + \ldots + D^{(2n-2)}X^{n-2}) \times (X^n - \beta) \equiv \left(D^{(0)} + \beta D^{(n)}\right) + \left(D^{(1)} + \beta D^{(n+1)}\right) B_1 + \ldots + D^{(n-1)} B_1^{n-1}$
- Perform Montgomery reduction to reduce the digits magnitude

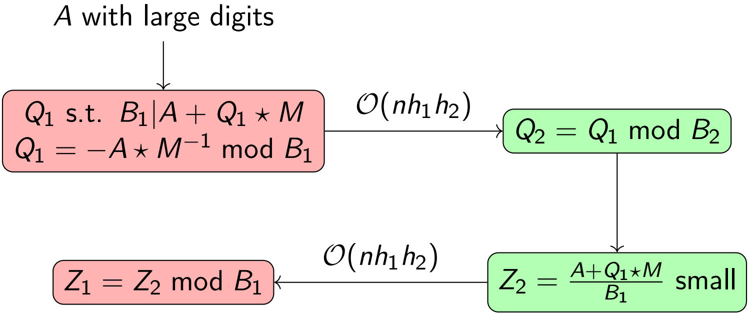
Hybrid Polynomial-Residue Number System

- Lattice $\mathcal{L}(\Gamma)$ of the representations of zero

$$\Gamma = \begin{bmatrix} P & 0 & \dots & 0 \\ -\gamma & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma^n & 0 & \dots & 1 \end{bmatrix}$$

- Each line in Γ corresponds to either $P = 0 \bmod P$ or $-\gamma^i + X^i$, which when evaluated at $X = \gamma$ produces a value congruent with 0
- Minskowski's theorem guarantees that $\mathcal{L}(\Gamma)$ contains a nonzero vector M of norm at most $(\det\mathcal{L}(\Gamma))^{1/n} = P^{1/n}$

Hybrid Polynomial-Residue Number System



$\star$ denotes multiplication in $\mathbb{Z}[X]/(X^n - \beta)$

Complexity dominated by $\mathcal{O}(n^2(h_1 + h_2) + nh_1 h_2)$ with $nh_1 \sim nh_2 \sim \log_2 P$

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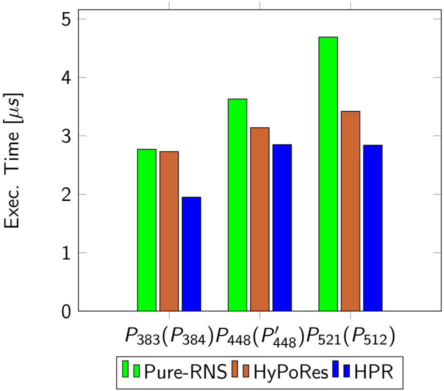
Experimental Results



Average execution time of a pure-RNS and the proposed approaches for standardised primes, as well as of HPR with specially crafted primes on a i7-3770K

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Protection against SCAs

- Choose γ as the root of $E(X) = E^{(0)} + \ldots + E^{(n-1)}X^{n-1} + X^n$
- Operate over $\mathbb{Z}[X]/(E(X))$ instead of $\mathbb{Z}[X]/(X^n - \beta)$
- Choose a E at random at the beginning of point multiplication
- Change representations throughout the execution of the algorithm by precomputing representations of γ^i in the target system

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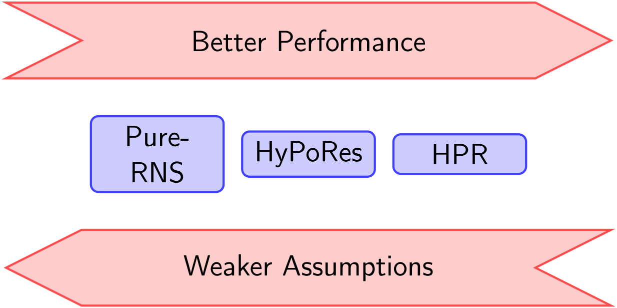
Conclusion



- HyPoRes multiplication has subquadratic time complexity
- Montgomery reduction is slower than carry propagation so HyPoRes is slower than HPR, but works for any prime
- Redundant representations are possible, improving resistance against SCAs

Thank you!

Any questions?